

HIGH STRENGTH CONCRETE AND LRFD LIVE LOAD DISTRIBUTION FACTORS

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ABSTRACT

The AASHTO Standard (LFD) live load distribution factors were developed over 50 years ago, and the AASHTO LRFD live load distribution factors were developed more than a decade ago. Each set was calibrated to the typical bridge parameters of its time. Changes in bridge design technology and advances in materials have resulted in differences in some common properties. For example, current practices allow designs with span lengths of over 300 ft, compared to 100 ft in older bridges. Similarly, high strength concrete (8 to 12 ksi) is utilized more commonly than the typical 6 ksi used a decade ago. A number of I-girder bridges have been analyzed using more accurate (finite element) methods and the results are compared with code specified live load distribution factors. Based on these results, recommendations are given for obtaining more accurate results for today's common bridge parameters.

Keywords: Bridge, Concrete, Spliced Girder, High Strength, HPC, Live Load, Load Distribution, LRFD, AASHTO

INTRODUCTION

The AASHTO Standard (LFD)¹ live load distribution factors were developed over 50 years ago, and the current AASHTO-LRFD² live load distribution factors are based on formulas developed by Zokaie, et al³ in 1991, and further modified for LRFD in 1994. At the time of this study, the average span length of a prestressed girder was 48 ft. Table 1 shows the range of values used in parameter studies that led to the current formulas. It was further decided to define ranges of applicability for the parameters in these formulas that correspond to the parameters used in the analysis and provide a level of comfort as to the accuracy of the formulas. Today's high strength materials and spliced girder technology allow the parameters to routinely exceed these ranges. When this occurs, the LRFD Specifications direct the designers to use the lever rule or perform a more detailed analysis. The lever rule has been shown to be highly conservative, and a more detailed analysis may not be feasible in all cases.

A number of hypothetical I-girder bridges have been analyzed using finite element methods and the results are compared with existing live load distribution factors. Based on these results, recommendations are given for obtaining more accurate results for today's common bridge parameters. Specifically, the effects of the following parameters on both moment and shear distribution factors are reviewed and discussed.

- The effect of concrete strength
- The effect of increased span length
- The effect of increased depth (and moment of inertia)
- The effect of large girder spacing
- A review of the limits of applicability of the LRFD live load distribution formulas

The following analytical study is intended to provide some understanding of the need for a more detailed analysis and the expected results so that designers can make a more informed decision.

TABLE 1. Parameter Values Used in Development of LRFD Specifications

Parameter	Parametric Value					
			Average			
Girder Spacing (ft)	4.5	6.04	7.77	9.5	13.1	-
Span Length (ft) (ft)	29.2	39.0	48.0	66.0	84.0	150.0
Bending Inertia, Kg [†] , (in ⁴)	398125	1448580	2499000	595700	9421300	-
Torsional Inertia, J, (in ⁴)	700	28000	182000	441000	700000	-
Slab Thickness (in)	-	6.0	6.95	9.0	-	-
Number of Girders	-	4	5	7	-	-
de* (ft)	0.0	0.05	2.5	3.0	4.5	6.5
Deck Strength, f'c (ksi)	-	-	4.0	-	-	-

[†] Kg = n(I+Ae²)

* de = Overhang – Curb Width

AASHTO SPECIFICATIONS

AASHTO STANDARD SPECIFICATIONS

The AASHTO Standard Specifications (LFD) live load distribution factor for moment is:

One lane: $S/7.0$ wheel lines = $S/14$ lanes: Use lever rule if $S > 10$ ft

Two or more lanes: $S/5.5$ wheel lines = $S/11$ lanes: Use lever rule if $S > 14$ ft

Where “S” is the girder spacing in ft

According to Section 3.23.1, for the shear distribution factor, the above moment formulas are also used for wheels that are away from the support locations, and the lever rule is used for wheels that are at support locations. The latter provision is generally ignored in design because, as for most shear response values, the wheels are not right at the support location.

AASHTO LRFD SPECIFICATIONS

The LRFD Specifications have formulas that try to capture the effect of other parameters such as span length and girder size. The LRFD moment formulas are as follows:

One lanes: $0.06 + (S/14)^{0.4} (S/L)^{0.3} (Kg/12.0Lts^3)^{0.1}$

Two or more lanes: $0.075 + (S/9.5)^{0.6} (S/L)^{0.2} (Kg/12.0Lts^3)^{0.1}$

Where: S is girder spacing in ft, $3.5 < S < 16$
 L is span length in ft, $20 < L < 240$
 ts is slab thickness in inches, $4.5 < ts < 12$
 $Kg = n(I + Ae^2)$ in in^4 , $10,000 < Kg < 7,000,000$
 n = Modular ratio of girder to deck
 I = girder inertia
 A = Girder Area
 e = distance from c.g. of girder to mid-height of slab

Live load distribution for shear is as follows:

One Lane: $0.36 + S/25$

Two or more lanes: $0.20 + S/12 - (S/35)^2$

Where S is girder spacing in ft, $3.5 < S < 16$

The LRFD Specifications also have adjustment factors for exterior girders and skewed support conditions. However, these conditions are not considered in this study.

THE PARAMETER STUDY

A number of bridges were analyzed using the grillage analysis procedure in the computer program LDFac⁴. The analytical study is performed by using a base (typical) bridge, and changing each of its parameters, one at a time, to study a broader range. Note that while one parameter is changed, all other parameters remain at the typical value. This allows us to study the effect of each parameter in an analytical way without concern for the cross-correlation that may exist among them. The base bridge in this case is a 150 ft long bridge made of 6 BT-72 girders at 8 ft spacing. The girders are made of 10 ksi concrete, and the 10 in. deck is made of 5 ksi concrete. The exterior girders have a 4 ft overhang with a curb width of 2.5 ft. Table 2 shows the list of parameters that were used for this study. Note that the slab thickness, concrete strength, and other parameters were selected to match typical values in modern bridges. The range of values on the lower end represent the typical bridge parameters of smaller bridges while at the upper end, they correspond to modern spliced girders, and in some cases looking to future values (e.g., f'c of 20 ksi.)

TABLE 2. Parameter Values Used in Parameter Study

Parameter	Parametric Value			
	Small	Typical	Large	Extended
Span Length (ft)	80	150	220	300
Girder Spacing (ft)	6.0	8.0	16.0	20.0
Girder Size	BT-54*	BT-72	BT-96*	BT-120*
Girder Strength, f'c (ksi)	6.0	10.0	15.0	20.0

*Same as BT-72, with an 8 in. web and modified depth to the size shown

SPAN LENGTH

Four span lengths are considered: 80 ft, 150 ft, 220 ft, and 300 ft. While the 80 ft span represents a traditional prestressed girder, 150 ft span is chosen as a design limit for the prestressed girder and the transition to spliced girders. The 220 ft span is the upper limit of a simply supported spliced girder, and the 300 ft is a typical span of a continuous spliced girder, or a simply supported spliced girder with very high strength concrete.

GIRDER SPACING

Four girder spacings are considered: 6 ft, 8 ft, 16 ft, and 20 ft. The 6 ft spacing represents a typical, small, prestressed girder, while the 8 ft spacing is a typical spacing for a larger prestressed girder. The 16 ft spacing is chosen as the upper limit of the current formulas in LRFD, and the 20 ft spacing is chosen as a futuristic value for illustrative purposes.

GIRDER SIZE

Four different girder sizes are considered: 54 in. deep, 72 in. deep, 96 in. deep, and 120 in. deep. The properties of the smaller girders coincide with BT-54 and BT-72 girders. The

larger girders are deeper sections with flanges similar to BT-72 and webs that are 8 in. wide to accommodate post-tensioning. The properties of these girders are shown in Table 3.

TABLE 3. Properties of Girders Used in Parameter Study

Girder Size	I (in⁴)	A (in²)	e (in)	Kg* (in⁴)
BT-54	268000	659	32	1329371
BT-72	546000	767	40	2500212
BT-96	1.19E+06	1087	50	5509575
BT-120	2.22E+06	1295	63	10377396

* Based on $f'c=10\text{ksi}$, i.e., $n=1.41$, modified for other cases based on girder $f'c$

CONCRETE STRENGTH

Four different concrete strengths are considered: 6, 10, 15, and 20 ksi. The strength value of 6 ksi represents a traditional prestressed girder, while 10 ksi represents a concrete strength that is becoming commonplace in precast construction. The strength of 15 ksi is currently achievable although not commonly specified, and the strength of 20 ksi is used here as a futuristic value for illustrative purposes.

The results of the parameter study are discussed below. Figs. 1, 3, 5, and 7 show the variation of the moment distribution factors as a factor of each of the bridge parameters. The same figures show the LFD and LRFD formula-based values as well. Figs. 2, 4, 6, and 8 show similar results based on shear distribution factors.

Figs. 1 through 8 provide a basic understanding of the effect of bridge parameters on the distribution factors; however, it is unreasonable to expect a 6 ft deep girder (e.g. BT-72) made of 10 ksi concrete to be sufficient for a 300 ft long span. For this reason, one more study was performed based on varying all bridge parameters at the same time. In this case, all small parameters are used together, followed by the next set, etc. That is to say, each case consists of parameters from each of the four columns in Table 2. The results of this study are shown in Figs. 9a and 9b. In addition, the ratio of the distribution factors from both code specifications to the more accurate analysis results is shown in Figs. 10a and 10b.

DISCUSSION OF RESULTS

In the following discussion, the effect of each parameter is reviewed first on both moment and shear distribution factors. The effect of each parameter is first discussed by reviewing the analytical results. Next, each of the LFD and LRFD results are reviewed to understand how well they predict these effects. As a guideline for discussion, it is assumed that code results that are up to 25 percent conservative are reasonable, and higher levels are too conservative. At the end, a review of the combined variation of all parameters is presented.

SPAN LENGTH

Fig. 1 shows the distribution factors for bridges with different span lengths. In this figure and other similar ones, M-2+ refers to moment distribution factor for two and more lanes loading, while M-1 refers to one lane loading, LFD, LRFD, and Anal refer to method used for calculating the distribution factors. These results indicate that moment distribution factors are smaller for longer bridge spans. While LFD ignores this fact, the LRFD Specifications do a relatively good job of predicting this effect. It can be seen that the LFD Specifications are a reasonable approximation for shorter spans, but they are highly conservative for larger spans. Furthermore, it seems that the limit of applicability of 240 ft set in the LRFD Specifications can be extended to 300 ft without any major concern. If the engineers are concerned about using this formula for bridges longer than 240 ft, then the upper limit of 240 ft can be used. This will be slightly conservative, compared to using the lever rule, which will be highly conservative. In this case, the lever rule will produce a multi-lane distribution factor of 0.875, which is much larger than a more accurate value of less than 0.5 obtained from analysis.

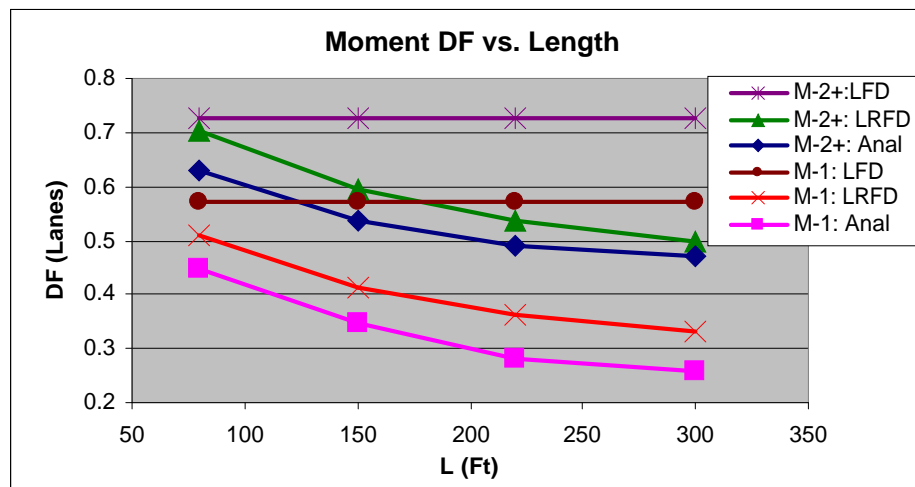


Fig. 1: Variation of Moment Distribution Factor with Span Length

Fig. 2 shows that varying span length does not have a major effect on the shear distribution factors. Both LFD and LRFD Specifications ignore the effect of span length. The LRFD Specifications produce a reasonable and conservative estimate. Note that the LFD values shown here are based on moment distribution values formulas, i.e., the provisions of Section 3.23.1 are ignored, as it is customary to do so. However, the LFD value is unconservative for all span ranges.

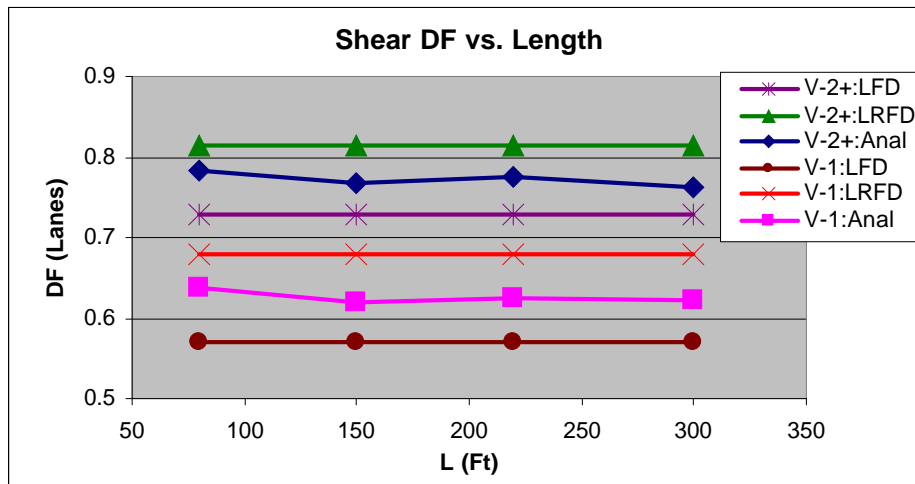


Fig. 2: Variation of Shear Distribution Factor with Span Length

Fig. 3 shows the variation of moment distribution factors with girder spacing. The distribution factor has a direct correlation with girder spacing as expected. The LRFD Specifications produce a reasonable estimate of these values even at the girder spacing of 20 ft, which is outside the range of applicability. Note that the LRFD values shown in Fig. 3 are based on direct application of the formula and ignore the limit of applicability. Therefore, the value for the 20 ft spacing is not based on the lever rule, but based on using an S of 20 ft in the formula. On the other hand, the LFD formula produces a reasonable to high estimate at the lower spacing values (e.g., 6 to 8 ft), but becomes highly conservative at higher values. Note that values shown for higher spacing are based on the lever rule, and the strict use of the formula will be even more conservative.

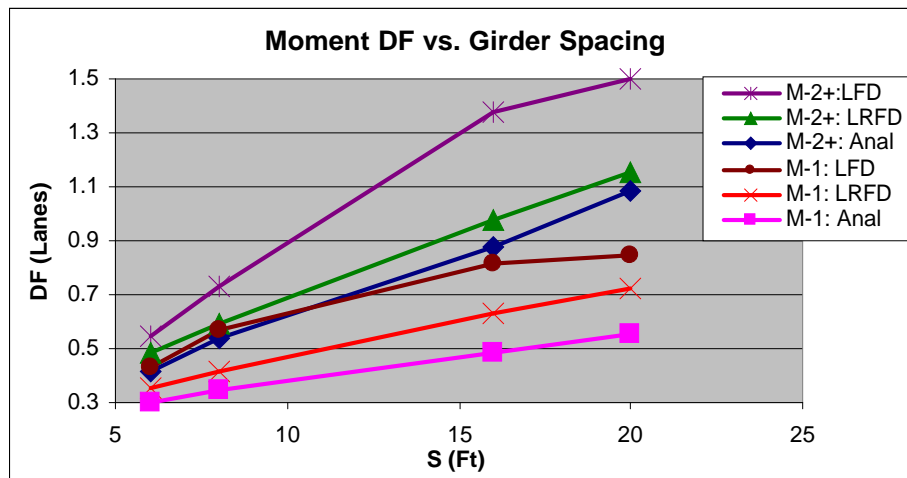


Fig. 3: Variation of Moment Distribution Factor with Girder Spacing

Fig. 4 shows the effect of girder spacing on shear distribution factors. Once again, the distribution factors have a direct correlation with girder spacing. Both LFD and LRFD formulas result in good and generally conservative values for the multi-lane factors, with the exception of LFD factor at 8 to 10 ft spacing. Furthermore, the LFD Specifications are

unconservative for single loading, and the LRFD Specifications become increasingly conservative. In this case, if the lever rule is used for the 20 ft spacing case with LRFD, the single lane factor will be 1.02 compared to a formula value of 1.16 and a grillage analysis value of 0.93; the LRFD multi-lane factor will be 1.5 compared to a formula value of 1.54 and grillage analysis value of 1.39.

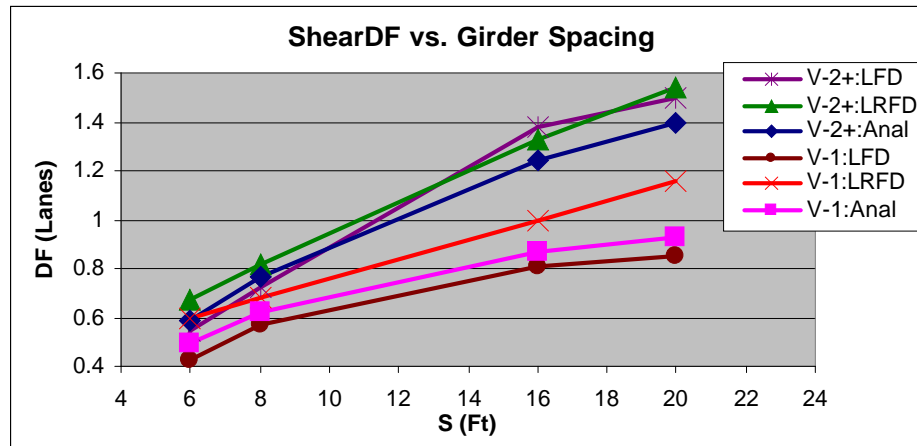


Fig. 4: Variation of Shear Distribution Factor with Girder Spacing

Fig. 5 shows the variation of moment distribution factor with girder size (depth and stiffness). This figure shows that increasing depth (and stiffness) causes a slight increase in the distribution factor. This effect is captured via the K_g parameter in LRFD and is ignored in the LFD Specifications. The LRFD Specifications produce a reasonable and conservative estimate at lower depth and stiffness values, but multi-lane estimates tend to get increasingly conservative for larger girders. Note that the formulas show more conservatism for deeper sections (8 ft and deeper). Also, it should be noted that the deepest section (10 ft) has a K_g value that is outside the range of applicability; however, although these results are more conservative, they are considerably less conservative than the lever rule. The LFD Specifications in this case produce highly conservative results (up 50 percent).

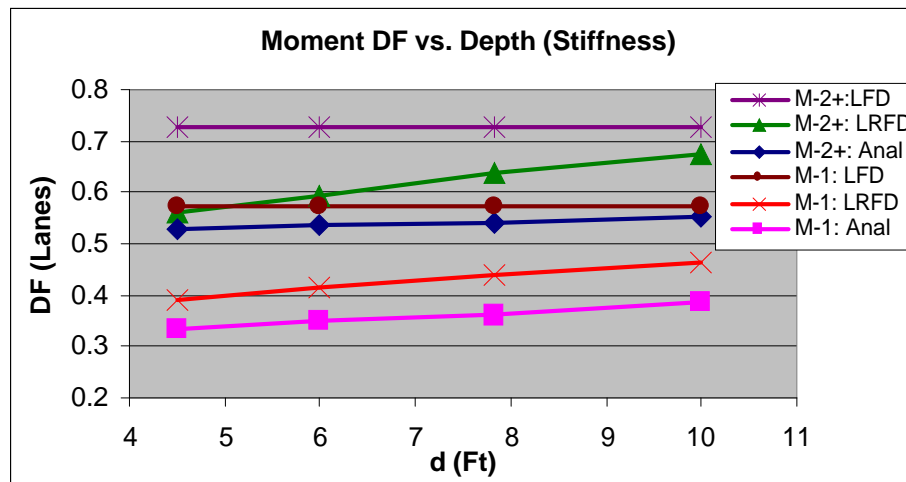


Fig. 5: Variation of Moment Distribution Factor with Girder Size

Fig. 6 shows the effect of girder size on the shear distribution factor. Once again, an increase in girder size causes an increase in the distribution factor. Both specifications ignore this effect. However, the LRFD Specifications remain slightly conservative, even at larger sizes (higher depth and stiffness), while LFD is unconservative for both single lane and multi-lane cases.

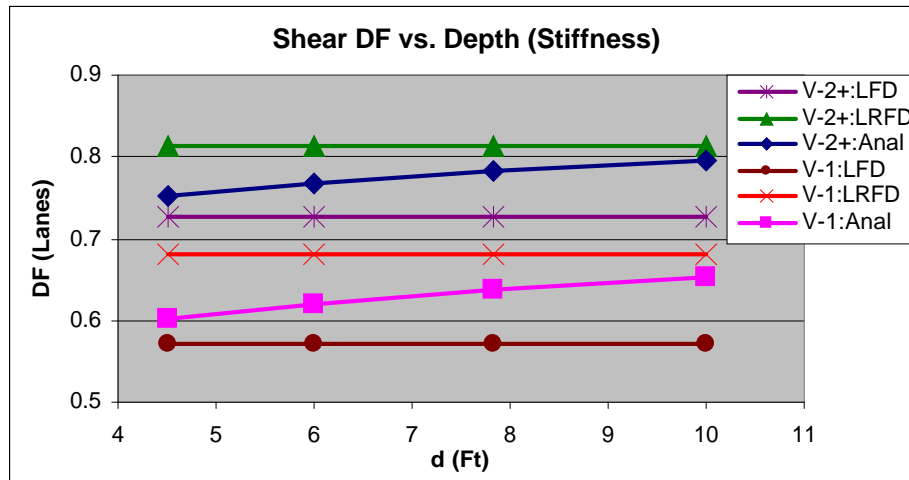


Fig. 6: Variation of Shear Distribution Factor with Girder Size

Fig. 7 shows the effect of concrete strength on moment distribution factors. The highest value considered in this case is 20 ksi, and although it is not a practical value at this time, it is expected that it might become routinely achievable in the near future. This figure shows that increasing concrete strength ($f'c$) has a very slight effect in increasing the analytical distribution factors. This effect is ignored in the LFD Specifications, and is captured via the Kg parameter (through the modular ratio, n) in LRFD. While the LRFD Specifications produce reasonable and conservative estimates, the LFD predictions are highly conservative.

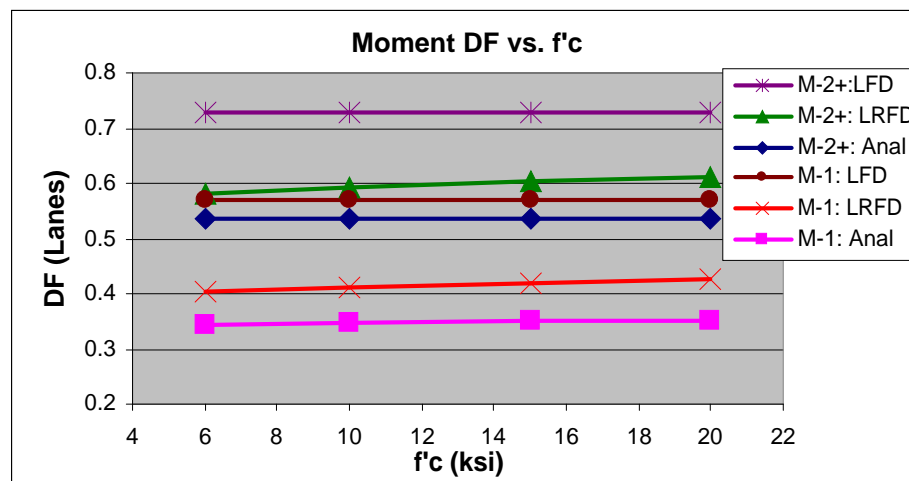


Fig. 7: Variation of Moment Distribution Factor with Concrete Strength

Fig. 8 shows the effect of concrete strength on shear distribution factors. Once again, this figure shows that the concrete strength has a negligible effect on the analytical shear distribution factor. This effect is ignored in both LFD and LRFD Specifications. The LRFD Specifications produce slightly conservative estimates while the LFD estimates are slightly unconservative.

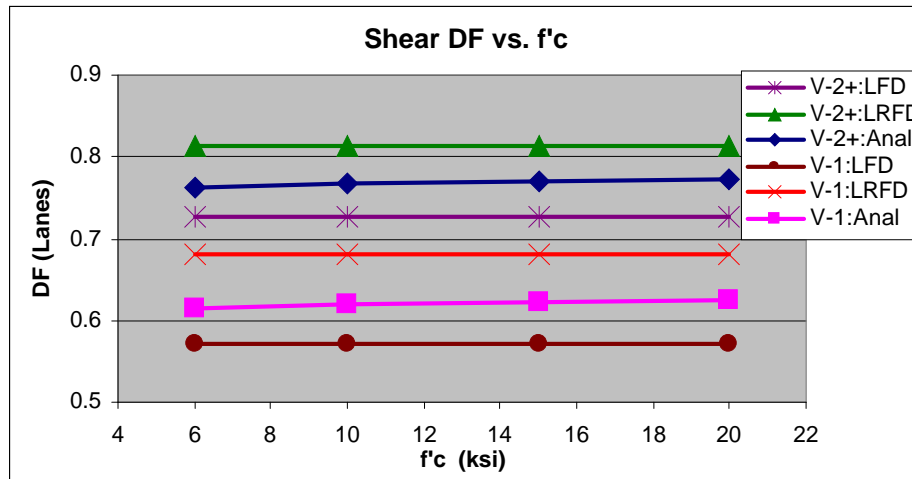


Fig. 8: Variation of Shear Distribution Factor with Concrete Strength

Fig. 9a shows the moment distribution factors for small to large bridges. The results are plotted against span length. However, the lower span length also represents lower spacing, lower girder size, and lower concrete strength. The higher span length also represents the higher spacing, size, and strength. The low end of the curve represents a 60 ft long span at 6 ft spacing using a BT-54 section made of 6 ksi concrete while the higher end represents a 300 ft span at 20 ft spacing, using a 10 ft deep section made of 20 ksi concrete. Fig. 9b shows the ratio of the formula values to grillage analysis.

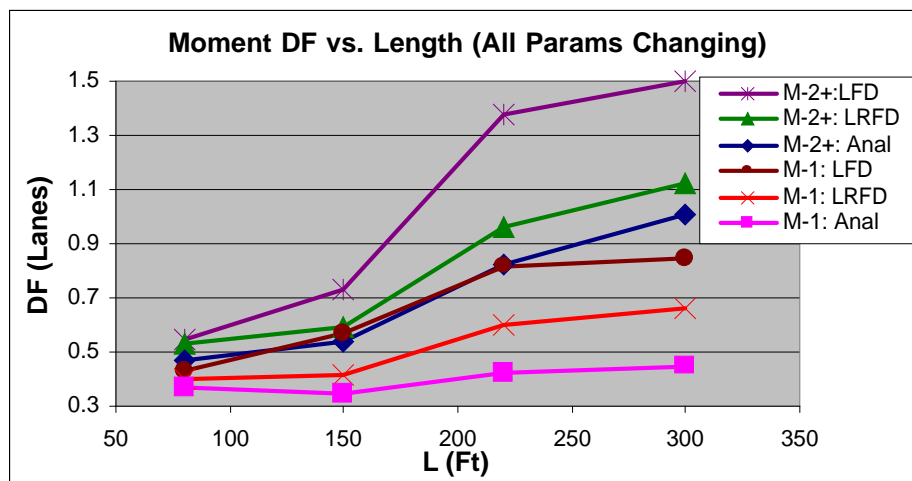


Fig. 9a: Variation of Moment Distribution Factor with Bridge Size

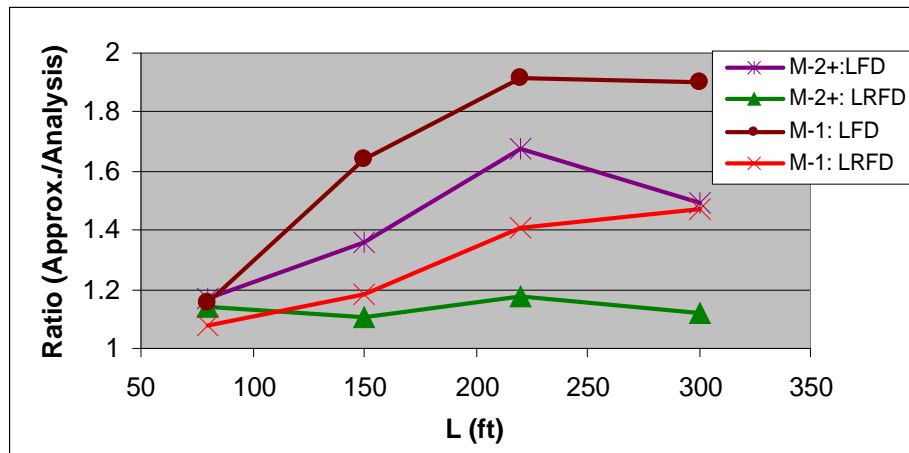


Fig. 9b: Ratio of Code to Analysis Moment Distribution Factor with Bridge Size

These figures show that, for a smaller bridge, both LFD and LRFD produce reasonable estimates. On the other hand, for larger bridges, for single lane loading, both specifications become increasingly conservative, with the LFD Specifications becoming highly and unreasonably conservative (nearly 90 percent). LFD estimates are highly conservative for the multi-lane loading as well. These figures show that the LRFD Specifications for multi-lane loading is the only case that produces reasonable results for large bridges. It should be noted that this case (multi-lane) is the typical case used in design.

Fig. 10a shows that the shear distribution factor for the same bridges shown in Fig. 9. Fig. 10b shows the ratios of the LFD and LRFD estimates to the grillage analysis. Once again, the LRFD Specifications produce slightly conservative estimates while the LFD Specifications are unconservative, with the exception of multi-lane loading for larger bridges.

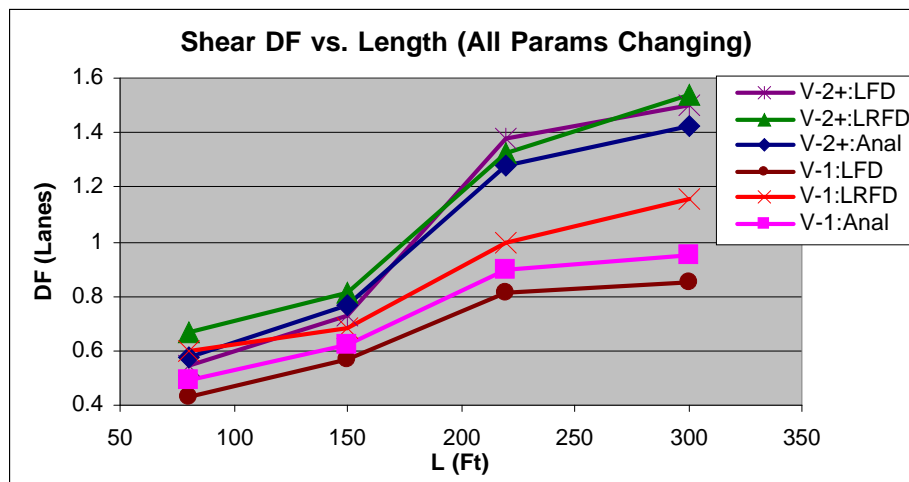


Fig. 10a: Variation of Shear Distribution Factor with Bridge Size

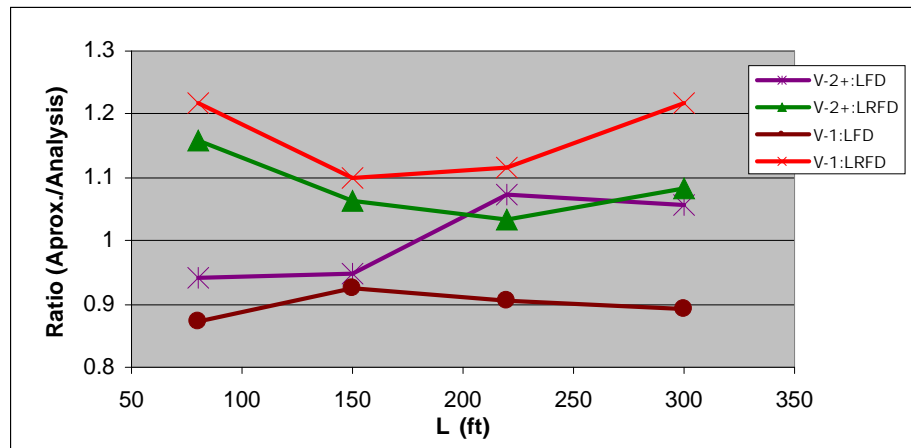


Fig. 10b: Ratio of Code to Analysis Shear Distribution Factor with Bridge Size

SUMMARY AND CONCLUSIONS

The parameter study performed and reported here extends our understanding of the live load distribution factors, especially as they relate to larger bridges. Concrete bridges continue to extend their range of use by using higher strength concrete, larger cross sections, and employing spliced girder technology to achieve higher span lengths. Therefore, it is important to clarify the guidelines for live load distribution in these cases. The following recommendations are made, partly based on the results of this study:

- If possible, use a more detailed analysis, such as a grillage analysis, to produce more accurate live load distribution factors.
- The limits of applicability of the formulas in the LRFD specifications can be extended without sacrificing the safety of the bridge. In most cases, extending these limits upward will result in more conservative predictions. Since spans over 300 ft were not evaluated, a span length of 300 can be used in the formula to remain conservative.
- The LFD moment distribution formulas are highly conservative for larger bridges.
- The LFD estimates of shear distribution tend to be unconservative when they are based on formulas for moment distribution.

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